snpt. x-as: $\cos(2t) = 0$

$2t = \frac{1}{2}\pi + k\pi$

$t = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$

dus voor $t = \frac{1}{4}\pi$ eerste snpt.

$\vec{v}(t) = \begin{pmatrix} -3\sin(2t) \\ -2\sin(2t) \end{pmatrix}$

$\vec{v}(\frac{1}{4}\pi) = \begin{pmatrix} -3 \cdot \frac{1}{2}\sqrt{2} \\ -2 \cdot 1 \end{pmatrix} = \begin{pmatrix} -1\frac{1}{2}\sqrt{2} \\ -2 \end{pmatrix}$

$|\vec{v}(\frac{1}{4}\pi)| = \sqrt{(-1\frac{1}{2}\sqrt{2})^2 + (-2)^2}$

$= \sqrt{4\frac{1}{2} + 4} = \sqrt{\frac{17}{2}} = \frac{1}{2}\sqrt{34}$

$\cos^2(\frac{1}{2}x) - \cos(\frac{1}{2}x) - 2 = 0$

$\cos(\frac{1}{2}x) = u$

$u^2 - u - 2 = 0$

$(u-2)(u+1) = 0$

$u = 2 \vee u = -1$

$\cos(\frac{1}{2}x) = 2 \vee \cos(\frac{1}{2}x) = -1$

geen opl.

$\frac{1}{2}x = \pi + k2\pi$

$x = 2\pi + k4\pi$

$25^x + 6 = 5^{x+1}$

$(5^2)^x + 6 = 5^x \cdot 5$

$(5^x)^2 - 5 \cdot 5^x - 6 = 0$

$5^x = u$

$u^2 - 5u - 6 = 0$

$(u-3)(u-2) = 0$

$u = 3 \vee u = 2$

$5^x = 3 \vee 5^x = 2$

$x = {}^5\log(3) \vee x = {}^5\log(2)$

De grafieken raken als

$\sqrt{5-x^2} = ax + 5 \wedge \frac{1}{2\sqrt{5-x^2}} \cdot -2x = a$

$\frac{-x}{\sqrt{5-x^2}} = a$

$\sqrt{5-x^2} = \frac{-x^2}{\sqrt{5-x^2}} + 5$

$5 - x^2 = -x^2 + 5\sqrt{5-x^2}$

$5 = 5\sqrt{5-x^2}$

$\sqrt{5-x^2} = 1$

$5 - x^2 = 1$

$x^2 = 4$

$x = 2 \vee x = -2$

$a = -2 \vee a = 2$

${}^3\log(x+1) = 4 + {}^{\frac{1}{3}}\log(x-1)$

${}^3\log(x+1) = {}^3\log(3^4) - {}^3\log(x-1)$

${}^3\log(x+1) = {}^3\log\left(\frac{81}{x-1}\right)$

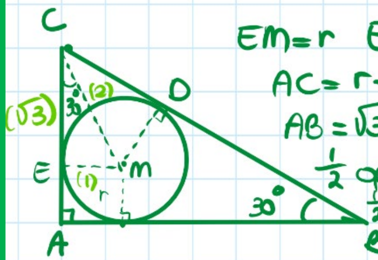
$x + 1 = \frac{81}{x-1}$

$x^2 - 1 = 81$

$x^2 = 82$

$x = \sqrt{82} \vee x = -\sqrt{82}$

vold. vold. niet



$EM = r \quad EC = r\sqrt{3}$

$AC = r + r\sqrt{3}$

$AB = \sqrt{3}\,AC = r\sqrt{3} + 3r$

$\frac{1}{2}$ opp $\Delta ABC =$

$\frac{1}{2} \cdot \frac{1}{2} \cdot (r + r\sqrt{3})(r\sqrt{3} + 3r) =$

$\frac{1}{4}(1+\sqrt{3})(\sqrt{3}+3)r^2 \approx$

$3{,}232r^2$

Opp. cirkel $= \pi r^2 \approx 3{,}141r^2$

dus opp. cirkel $< \frac{1}{2} \cdot$ opp ΔABC

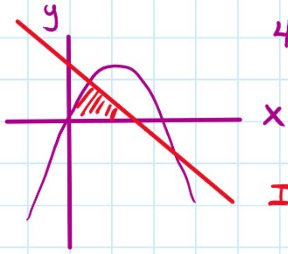
$$\cos\left(2x-\tfrac{1}{2}\pi\right)=\cos\left(x+\tfrac{1}{3}\pi\right)$$

$$2x-\tfrac{1}{2}\pi = x+\tfrac{1}{3}\pi + k2\pi \ \vee$$

$$2x-\tfrac{1}{2}\pi = -x-\tfrac{1}{3}\pi + k2\pi$$

$$x=\tfrac{5}{6}\pi + k2\pi \ \vee \ 3x=\tfrac{1}{6}\pi + k2\pi$$

$$x=\tfrac{5}{6}\pi + k2\pi \ \vee \ x=\tfrac{1}{18}\pi + k\tfrac{2}{3}\pi$$



$$4x-x^2=5-2x \qquad 5-2x=0$$

$$x^2-6x+5=0 \qquad 2x=5$$

$$(x-5)(x-1)=0 \qquad x=2\tfrac{1}{2} \qquad g(1)=3$$

$$x=1 \ \vee x=5$$

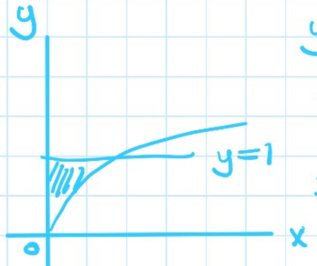
$$I(\mathcal{L})=\pi\int_0^1\left(4x-x^2\right)^2dx+\tfrac{1}{3}\cdot\pi\cdot 3^2\left(2\tfrac{1}{2}-1\right)$$

$$=\pi\int_0^1\left(16x^2-8x^3+x^4\right)dx+4\tfrac{1}{2}\pi$$

$$=\pi\left[\tfrac{16}{3}x^3-2x^4+\tfrac{1}{5}x^5\right]_0^1+4\tfrac{1}{2}\pi$$

$$=\pi\left(\tfrac{16}{3}-2+\tfrac{1}{5}\right)+4\tfrac{1}{2}\pi$$

$$=8\tfrac{1}{30}\pi$$



$$y=\ln\left(\tfrac{1}{2}x+1\right)$$

$$\tfrac{1}{2}x+1=e^y$$

$$\tfrac{1}{2}x=e^y-1$$

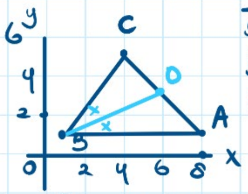
$$x=2e^y-2$$

$$I(\mathcal{L})=\pi\int_0^1\left(2e^y-2\right)^2dy=$$

$$\pi\int_0^1\left(4e^{2y}-8e^y+4\right)dy=$$

$$\pi\left[2e^{2y}-8e^y+4y\right]_0^1=\pi\left(2e^2-8e+4-2+8\right)$$

$$=\pi\left(2e^2-8e+10\right)$$



$$\vec{BA}=\vec{a}-\vec{b}=\begin{pmatrix}8\\1\end{pmatrix}-\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}7\\0\end{pmatrix} \quad |\vec{BA}|=7$$

$$\vec{BC}=\vec{c}-\vec{b}=\begin{pmatrix}4\\5\end{pmatrix}-\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}3\\4\end{pmatrix} \quad |\vec{BC}|=5$$

$$\vec{BD}=5\cdot\begin{pmatrix}7\\0\end{pmatrix}+7\cdot\begin{pmatrix}3\\4\end{pmatrix}=\begin{pmatrix}35\\0\end{pmatrix}+\begin{pmatrix}21\\28\end{pmatrix}$$

$$=\begin{pmatrix}56\\28\end{pmatrix}\triangleq\begin{pmatrix}2\\1\end{pmatrix}$$

$$BD:\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\1\end{pmatrix}$$

$$\vec{AC}=\vec{c}-\vec{a}=\begin{pmatrix}-4\\4\end{pmatrix}\triangleq\begin{pmatrix}-1\\1\end{pmatrix} \text{ dus } \vec{n}_{AC}=\begin{pmatrix}1\\1\end{pmatrix}$$

$$\left.\begin{array}{l}AC: x+y=c\\ \text{door } (8,1)\end{array}\right\} AC: x+y=9$$

dus $D\left(5\tfrac{2}{3}, 3\tfrac{1}{3}\right)$

snpt. BD & AC:

$$1+2\lambda+1+\lambda=9$$

$$3\lambda=7 \quad \lambda=2\tfrac{1}{3}$$

$$f(x)=16(4x+10)^{-5} \qquad F(x)=-\tfrac{16}{4}(4x+10)^{-4}\cdot\tfrac{1}{4}+c$$

$$=\frac{-1}{(4x+10)^4}+c$$

$$g(x)=\cos(\pi(x-3)) \qquad G(x)=\tfrac{1}{\pi}\sin(\pi(x-3))+c$$

Grafieken snijden ⊥ als

$$\ln(x)=x^2+px \ \wedge \ \tfrac{1}{x}\cdot(2x+p)=-1$$

$$2x+p=-x$$

$$3x=-p$$

$$\ln(x)=x^2-3x^2$$

$$\ln(x)=-2x^2$$

$\uparrow$ y_1 $\quad$ $\uparrow$ y_2

optie intersect: $x\approx 0{,}55$

dus $p=-3x\approx-1{,}64$

$f_a(x) = (x+a)\ln(x)$

$f_a'(x) = \ln(x) + \frac{x+a}{x}$

$= \ln(x) + 1 + ax^{-1}$

$f_a''(x) = \frac{1}{x} - ax^{-2}$

$= \frac{1}{x} - \frac{a}{x^2}$

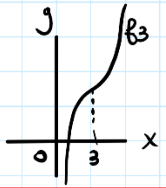
$f_a''(3) = 0$ geeft

$\frac{1}{3} - \frac{a}{9} = 0$

$\frac{1}{3} = \frac{a}{9}$

$3a = 9$

$a = 3$



vert. as. noemer = 0 ∧ teller ≠ 0

$1-2e^x = 0 \wedge e^x + 1 \neq 0$

$e^x = \frac{1}{2} \wedge e^x \neq -1$

$x = \ln(\frac{1}{2})$

hor. as. $\lim_{x\to\infty} \frac{e^x+1}{1-2e^x} = \lim_{x\to\infty} \frac{1+\frac{1}{e^x}}{\frac{1}{e^x}-2} = \frac{1+0}{0-2} = -\frac{1}{2}$

$\lim_{x\to-\infty} \frac{e^x+1}{1-2e^x} = \frac{0+1}{1-0} = 1$ dus $y = -\frac{1}{2} \wedge y = 1$

$2 + \frac{3}{4x} = \frac{a}{y}$ $\quad x=3$ geeft $y=4$

$2 + \frac{3}{12} = \frac{a}{4}$ $\quad \frac{8x+3}{4x} = \frac{9}{y}$

$2\frac{1}{4} = \frac{a}{4}$ dus $a = 9$

$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{36x}{8x+3} = \lim_{x\to\infty} \frac{36}{8+\frac{3}{x}} = \frac{36}{8+0} = 4\frac{1}{2}$

$y = 2 + 3^{0,2x-1}$

$2 + 3^{0,2x-1} = y$

$3^{0,2x-1} = y-2$

$0,2x - 1 = {}^3\log(y-2)$

$0,2x = {}^3\log(y-2) + 1$

$x = 5 \cdot {}^3\log(y-2) + 5$

c: $(x-3)^2 - 9 + (y+2)^2 - 4 + 5 = 0$

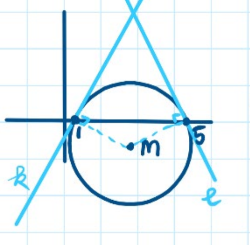
$(x-3)^2 + (y+2)^2 = 8$ $\quad M(3,-2)$ $\quad r = \sqrt{8}$

snpt. c met x-as: $(x-3)^2 + 4 = 8$

$(x-3)^2 = 4$

$x-3 = 2 \vee x-3 = -2$

$x = 5 \vee x = 1$



$\vec{r}_k = \binom{2}{-2}_\perp = \binom{2}{2} \triangleq \binom{1}{1}$

$\vec{r}_l = \binom{-2}{-2}_\perp = \binom{2}{-2} \triangleq \binom{1}{-1}$

omdat $\vec{r}_k \perp \vec{r}_l$

geldt $\angle(k,l) = 90°$

De grafiek van f is lijnsymm. in $x = \frac{1}{2}\pi$ als

$f(\frac{1}{2}\pi - p) = f(\frac{1}{2}\pi + p)$ voor alle p, dus als

$2 + (\frac{1}{2}\pi - p - \frac{1}{2}\pi)\cos(\frac{1}{2}\pi - p) = 2 + (\frac{1}{2}\pi + p - \frac{1}{2}\pi)\cos(\frac{1}{2}\pi + p)$

$2 - p\cos(\frac{1}{2}\pi - p) = 2 + p\cos(\frac{1}{2}\pi + p)$

$2 - p\cos(-\frac{1}{2}\pi + p) = 2 - p\cos(-\frac{1}{2}\pi + p)$

$\cos(A) = \cos(-A)$

$\cos(A) = -\cos(A - \pi)$

$1+\tan(\frac{1}{2}x-\frac{1}{3}\pi)=2$

$\tan(\frac{1}{2}x-\frac{1}{3}\pi)=1$

$\frac{1}{2}x-\frac{1}{3}\pi=\frac{1}{4}\pi+k\pi$

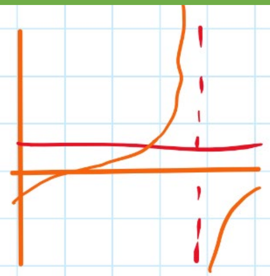
$\frac{1}{2}x=\frac{7}{12}\pi+k\pi$

$x=1\frac{1}{6}\pi+k2\pi$

periode $=\frac{\pi}{\frac{1}{2}}=2\pi$

asymptoot: $x=\frac{2}{3}\pi+\pi=1\frac{2}{3}\pi$

dus $f(x)>2$ geeft $1\frac{1}{6}\pi<x<1\frac{2}{3}\pi$



raaklijn // x-as:

$y'(t)=0 \;\wedge\; x'(t)\neq 0$

$2t=0 \;\wedge\; 2t^2-4t-6\neq 0$

$t=0 \;\wedge\; 0-0-6\neq 0$

$x(0)=0$ dus $(0,-4)$

$y(0)=-4$

$k: \frac{x}{2p}+\frac{y}{p}=1$ ofwel $x+2y=2p$ door $(6,1)$ $\Big\}$ $6+2=2p$, $p=4$

$c: (x+2)^2-4+(y-3)^2-9+8=0$

$(x+2)^2+(y-3)^2=5$

$m(-2,3)$ $r=\sqrt{5}$

$d(A,m)=\sqrt{(2--2)^2+(11-3)^2}$

$=\sqrt{16+64}=\sqrt{80}=4\sqrt{5}$

Dus $d(A,c)=4\sqrt{5}-\sqrt{5}=3\sqrt{5}$

$f(x)=\cos^2(4x)+\sin^2(6x)$

$=\frac{1}{2}+\frac{1}{2}\cos(8x)+\frac{1}{2}-\frac{1}{2}\cos(12x)$

$=1+\frac{1}{2}\cos(8x)-\frac{1}{2}\cos(12x)$

$F(x)=x+\frac{1}{16}\sin(8x)-\frac{1}{24}\sin(12x)+c$

$x(t)=\cos(t)$

$y(t)=2\sin(2t-\frac{1}{2}\pi)$ $\Big\}$ $y=2-4x^2$

$2\sin(2t-\frac{1}{2}\pi)=2-4\cos^2 t)$

$\sin(2t-\frac{1}{2}\pi)=1-2\cos^2(t)$

$\cos(2t-\frac{1}{2}\pi+\frac{1}{2}\pi)=\cos(2t)$

$\cos \quad -)=\cos(2t)$

en bovendien geldt $-1\leq x(t)\leq 1$

$$2\cos^2(x)+\sin(x)-2=0$$
$$2(1-\sin^2(x))+\sin(x)-2=0$$
$$2-2\sin^2(x)+\sin(x)-2=0$$
$$-2\sin^2(x)+\sin(x)=0$$
$$-\sin(x)(\sin(x)-1)=0$$
$$\sin(x)=0 \vee \sin(x)=1$$
$$x=k\pi \vee x=\tfrac{1}{2}\pi+k2\pi$$

$$e^{2x}-6e^{x+1}+5e^2=0$$
$$e^{2x}-6e\cdot e^x+5e^2=0$$
$$e^x=u$$
$$u^2-6eu+5e^2=0$$
$$(u-5e)(u-e)=0$$
$$u=5e \vee u=e$$
$$e^x=5e \vee e^x=e$$
$$x=\ln(5e) \vee x=1$$

$$y=\sin(x) \xrightarrow{V_{y,2}} y=\sin(\tfrac{1}{2}x) \xrightarrow{T(-2,\frac{1}{4}\pi)} y=\sin(\tfrac{1}{2}(x+2))+\tfrac{1}{4}\pi$$

$$f(x)=\frac{4x-3}{2x+1}=\frac{2(2x+1)-5}{2x+1}$$
$$=2-\frac{5}{2x+1}$$
$$F(x)=2x-5\cdot\ln|2x+1|\cdot\tfrac{1}{2}+c$$
$$=2x-2\tfrac{1}{2}\ln|2x+1|+c$$

$$y=\frac{x-4}{x-6}$$
$$x=\frac{y-4}{y-6}$$
$$xy-6x=y-4$$
$$xy-y=6x-4$$
$$y(x-1)=6x-4$$
$$y=\frac{6x-4}{x-1}=\frac{5(x-1)+x+1}{x-1}$$
$$=5+\frac{x+1}{x-1} \quad \text{dus } a=5$$

$$f_a(x)=\frac{x^2+4x-5}{2x+a}=\frac{(x+5)(x-1)}{2(x+\frac{1}{2}a)}$$
dus $\frac{1}{2}a=5 \vee \frac{1}{2}a=-1$ ofwel
$$a=10 \vee a=-2$$

$$f(x)=\frac{x^3+5x^2+6x+4+\sqrt{x}}{x^2}=x+5+6x^{-1}+4x^{-2}+x^{-1\frac{1}{2}}$$

$$F(x)=\tfrac{1}{2}x^2+5x+6\ln|x|-4x^{-1}-2x^{-\frac{1}{2}}+c$$

$$=\tfrac{1}{2}x^2+5x+6\ln|x|-\frac{4}{x}-\frac{2}{\sqrt{x}}+c$$

$$g_{jr}^{103}=\tfrac{1}{2}\qquad g_{jr}=\sqrt[103]{\tfrac{1}{2}}$$

$$\approx 0{,}993$$

Dus de hoeveelheid neemt jaarlijks met 0,67 % af

$$\text{e.s.}=\frac{5+-1}{2}=2=a$$

amplitude $=|b|=3$

$\frac{3}{4}$ periode $=\pi\rightarrow$ periode $=\frac{4}{3}\pi$

dus $C=\dfrac{2\pi}{4/3\pi}=1\tfrac{1}{2}$

beginpunt: $x=0+\frac{2}{3}\pi=\frac{2}{3}\pi$

dus $y=2+3\cos(1\tfrac{1}{2}(x-\tfrac{2}{3}\pi))$

$k: 2x+3y=12\quad \vec{n}_k=\binom{2}{3}\quad \vec{r}_k=\binom{3}{-2}\quad rc_k=-\frac{2}{3}$

$$\ell:\ \begin{cases} x=-3t-10 & |7| \\ y=7t+2 & |3| \end{cases}\qquad \begin{array}{l} 7x=-21t-70 \\ 3y=21t+6 \end{array}\ +$$

$$\ell:\ 7x+3y=-64$$

$\vec{n}_\ell=\binom{7}{3}\quad \vec{r}_\ell=\binom{3}{-7}\quad rc_\ell=-2\frac{1}{3}$

$$\left.\begin{array}{ll} \tan(\alpha)=-\frac{2}{3} & \alpha\approx-33{,}7^\circ \\ \tan(\beta)=-2\frac{1}{3} & \beta\approx-66{,}8^\circ \end{array}\right\}\ \varphi=\alpha-\beta\approx 33{,}1^\circ$$

$$f(x)=\frac{x^3+3x^2-5x+8}{x^2-1}=\frac{x(x^2-1)+x+3x^2-5x+8}{x^2-1}$$

$$=x+\frac{3x^2-4x+8}{x^2-1}=x+\frac{3(x^2-1)+3-4x+8}{x^2-1}$$

$$=x+3+\frac{11-4x}{x^2-1}\qquad \lim_{x\to\pm\infty}\frac{11-4x}{x^2-1}=\lim_{x\to\pm\infty}\frac{\frac{11}{x^2}-\frac{4}{x}}{1-\frac{1}{x^2}}$$

$$=\frac{0-0}{1-0}=0$$

dus schuine as. $y=x+3$

vert. as: $x^2-1=0 \wedge x^3+3x^2-5x+8\neq 0$

$x^2=1$

$x=1 \vee x=-1$

$$y={}^5\log(x)\xrightarrow{T(3,-2)}y=-2+{}^5\log(x-3)\xrightarrow{V_{y,\frac{1}{2}}}y=-2+{}^5\log(2x-3)$$

$\cos(\alpha)=\frac{1}{2} \rightarrow \alpha=\frac{1}{3}\pi$

dus $\beta=\frac{1}{3}\pi+\frac{1}{2}\pi=\frac{5}{6}\pi \quad B(-\frac{1}{2}\sqrt{3},\frac{1}{2})$

$\gamma=\frac{5}{6}\pi+\frac{1}{2}\pi=1\frac{1}{3}\pi \quad C(-\frac{1}{2},-\frac{1}{2}\sqrt{3})$

$\delta=1\frac{1}{3}\pi+\frac{1}{2}\pi=1\frac{5}{6}\pi \quad D(\frac{1}{2}\sqrt{3},-\frac{1}{2})$

$\ln^2(x)-8\ln(x)+12=0$

$\ln(x)=u$

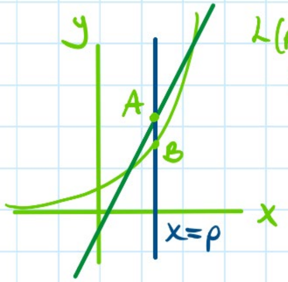
$u^2-8u+12=0$

$(u-6)(u-2)=0$

$u=6 \vee u=2$

$\ln(x)=6 \vee \ln(x)=2$

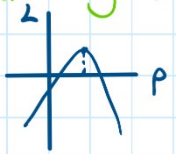
$x=e^6 \vee x=e^2$



$L(p)=g(p)-f(p)=3x-1-e^{2p-3}$

$f'(p)=3-2e^{2p-3}$

$f'(p)=0$ geeft $2e^{2p-3}=3$



$e^{2p-3}=1\frac{1}{2}$

$2p-3=\ln(1\frac{1}{2})$

$2p=3+\ln(1\frac{1}{2})$

$p=1\frac{1}{2}+\frac{1}{2}\ln(1\frac{1}{2})$

$L_{max}=L(1\frac{1}{2}+\frac{1}{2}\ln(1\frac{1}{2}))=3(1\frac{1}{2}+\frac{1}{2}\ln(1\frac{1}{2}))-1-e^{\ln(1\frac{1}{2})}$

$=4\frac{1}{2}+1\frac{1}{2}\ln(1\frac{1}{2})-1-1\frac{1}{2}=2+1\frac{1}{2}\ln(1\frac{1}{2})$

$a(t)=0{,}12t^2$

$v(t)=0{,}04t^3+c, \quad v(0)=0$ dus $c=0$

$s(t)=0{,}01t^4 \quad s(0)=0 \quad v(5)=0{,}04\cdot 5^3=5$

na 10 sec. is $s(5)+5\cdot 5=0{,}01\cdot 5^4+25$

$=31{,}25$ m.

$f_a(x)=\dfrac{\ln(ax^2)+2}{\ln(x)+4}$ perforatie als

$\ln(ax^2)+2=0 \wedge \ln(x)+4=0$

$\ln(ax^2)=-2 \vee \ln(x)=-4$

$ax^2=e^{-2} \wedge x=e^{-4}$

$a\cdot(e^{-4})^2=e^{-2}$

$ae^{-8}=e^{-2}$

$a=\dfrac{e^{-2}}{e^{-8}}=e^6$

$\vec{v}(t)=\begin{pmatrix}\cos(t-\frac{1}{4}\pi)\\ 2\cos(2t)\end{pmatrix}$

snpt. baan met x-as: $\sin(2t)=0$

$2t=k\pi$

$t=k\cdot\frac{1}{2}\pi$

$t=0$ geeft $x=\sin(-\frac{1}{4}\pi)=-\frac{1}{2}\sqrt{2}$

$t=\frac{1}{2}\pi$ geeft $x=\sin(\frac{1}{4}\pi)=\frac{1}{2}\sqrt{2}$

$t=1\frac{1}{2}\pi$ geeft $x=\sin(1\frac{1}{4}\pi)=-\frac{1}{2}\sqrt{2}$

$t=2\pi$ geeft $x=\sin(1\frac{3}{4}\pi)=\frac{1}{2}\sqrt{2}$

$\vec{v}(\frac{1}{2}\pi)=\begin{pmatrix}\cos(\frac{1}{4}\pi)\\ 2\cos(\pi)\end{pmatrix}=\begin{pmatrix}\frac{1}{2}\sqrt{2}\\ -2\end{pmatrix}$

rico raaklijn $=\dfrac{-2}{\frac{1}{2}\sqrt{2}}=-2\sqrt{2}$

$\tan(\alpha)=-2\sqrt{2}$ geeft

$\alpha=-70{,}52\ldots^\circ$

dus gevraagde hoek is 71°