Uitwerkingen examenopdrachten goniometrie

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3A

$x'(t) = -3\cos^2(t)\sin(t)$

$y'(t) = 3\sin^2(t)\cos(t)$

$\text{helling} = \frac{y'(t)}{x'(t)} = \frac{3\sin^2(t)\cos(t)}{-3\cos^2(t)\sin(t)} = -\frac{\sin(t)}{\cos(t)}$

3B

$y = -\frac{\sin(t)}{\cos(t)} \cdot x + b$ door $(\cos^3(t), \sin^3(t))$

$\sin^3(t) = -\frac{\sin(t)}{\cos(t)} \cdot \cos^3(t) + b$

$\sin^3(t) = -\sin(t)\cos^2(t) + b$

$b = \sin^3(t) + \sin(t)\cos^2(t)$

$= \sin^3(t) + \sin(t)(1-\sin^2(t))$

$= \sin(t)$

dus $y = -\frac{\sin(t)}{\cos(t)} \cdot x + \sin(t)$

3C

snpt. y-as: $x=0$ geeft $y = \sin(t)$

snpt. x-as: $y=0$ geeft $\frac{\sin(t)}{\cos(t)} \cdot x = \sin(t)$

$x = \cos(t)$

dus $A(0, \sin(t))$ en $B(\cos(t), 0)$

$AB^2 = (0-\cos(t))^2 + (\sin(t)-0)^2$

$= \sin^2(t) + \cos^2(t) = 1$

dus $AB = 1 \ \forall t$

3D

$f(x) = \frac{1}{2}(\sin(3x) - \sin(x))$

$= \frac{1}{2}(\sin(2x+x) - \sin(2x-x))$

$= \frac{1}{2}(\sin(2x)\cos(x) + \cos(2x)\sin(x)$

$-(\sin(2x)\cos(x) - \cos(2x)\sin(x)))$

$= \frac{1}{2}(\cos(2x)\sin(x) + \cos(2x)\sin(x))$

$= \sin(x)\cos(2x)$

4A

P op y-as: $y(t) = 0$ dus

$\sin(2t) = \sin(t)$

$2t = t + k2\pi \ \vee \ 2t = \pi - t + k2\pi$

$t = k2\pi \ \vee \ 3t = \pi + k2\pi$

$t = \frac{1}{3}\pi + k \cdot \frac{2}{3}\pi$

t op $[0, 2\pi]$ geeft $t = 0 \vee t = \frac{1}{3}\pi \ \vee$

$t = \pi \vee t = 1\frac{1}{3}\pi \ \vee t = 2\pi$

$x(\frac{1}{3}\pi) = \cos(\frac{2}{3}\pi) - \sin(\frac{2}{3}\pi)$

$= -\frac{1}{2} - \frac{1}{2}\sqrt{3} < 0$ dus

de x-coördinaat op dat tijdstip is $-\frac{1}{2} - \frac{1}{2}\sqrt{3}$

4B

$x = \pi$ is verticale asymptoot als

$1 - 2\cos(ax) = 0 \ \wedge \ \sin(ax) \neq 0$ met $x = \pi$

$1 - 2\cos(a\pi) = 0 \ \wedge \ \sin(a\pi) \neq 0$

$2\cos(a\pi) = 1 \ \wedge \ a\pi \neq k\pi$

$\cos(a\pi) = \frac{1}{2} \ \wedge \ a \neq k$

$(a\pi = \frac{1}{3}\pi + k2\pi \ \vee \ a\pi = -\frac{1}{3}\pi + k2\pi) \wedge a \neq k$

$(a = \frac{1}{3} + k \cdot 2 \ \vee \ a = -\frac{1}{3} + k \cdot 2) \ \wedge \ a \neq k$

dus $a = \frac{1}{3} + k \cdot 2 \ \vee \ a = -\frac{1}{3} + k \cdot 2$

4C

AB loopt verticaal, dus lijn door AB is $y = 1$

snpt. $f(x)$ en $y=1$ geeft

$2\sin(x - \frac{1}{3}\pi) = 1$

$\sin(x - \frac{1}{3}\pi) = \frac{1}{2}$

$x - \frac{1}{3}\pi = \frac{1}{6}\pi + k2\pi \ \vee \ x - \frac{1}{3}\pi = \frac{5}{6}\pi + k2\pi$

$x = \frac{1}{2}\pi + k2\pi \ \vee \ x = 1\frac{1}{6}\pi + k2\pi$

x op $[1\frac{1}{2}\pi, 16\pi]$ geeft $x = 12\frac{1}{2}\pi \ \vee$

$x = 3\frac{1}{6}\pi \ \vee \ x = 14\frac{1}{2}\pi \ \vee \ x = 15\frac{1}{6}\pi$

4D

snpt. x-as: $-2\cos^2(x) + 3 \cdot \cos(x) - 1 = 0$

$2\cos^2(x) - 3\cos(x) + 1 = 0$

$D = 9 - 4 \cdot 2 \cdot 1 = 1$

$\cos(x) = \frac{3 \pm 1}{4}$

$\cos(x) = 1 \ \vee \ \cos(x) = \frac{1}{2}$

$x = k2\pi \ \vee \ x = \frac{1}{3}\pi + k2\pi \ \vee$

$x = -\frac{1}{3}\pi + k2\pi$

$0 \leq x \leq \frac{1}{2}\pi$ geeft $x = 0 \ \vee \ x = \frac{1}{3}\pi$

dus $x = \frac{1}{3}\pi$

5A De grafiek van $f_2(x)$ is puntsymm. in $(\frac{1}{2}\pi, 0)$ als

$$f_2(\tfrac{1}{2}\pi - p) + f_2(\tfrac{1}{2}\pi + p) = 0 \quad \forall p$$

dus
$$\frac{\sin(\pi-2p)}{1-2\cos(\pi-2p)} + \frac{\sin(\pi+2p)}{1-2\cos(\pi+2p)} = 0$$

$$\frac{-\sin(-\pi+2p)}{1-2\cos(-\pi+2p)} + \frac{\sin(\pi+2p)}{1-2\cos(\pi+2p)} = 0$$

$$\frac{-\sin(\pi+2p)}{1-2\cos(\pi+2p)} + \frac{\sin(\pi+2p)}{1-2\cos(\pi+2p)} = 0$$

5B $f'(x) = -6\sin(2x) - \frac{1}{\sqrt{2x}}$

$f'(x) = 0$ geeft $-6\sin(2x) - \frac{1}{\sqrt{2x}} = 0$

$-6\sin(2x) = \frac{1}{\sqrt{2x}}$

$y_1 = -6\sin(2x)$ window: $[0,10] \times [\quad]$

$y_2 = \frac{1}{\sqrt{2x}}$

geeft $x_B \approx 4{,}739$

snpt. g met x-as: $3 - \sqrt{2x} = 0$

$\sqrt{2x} = 3$

$2x = 9$

$x_A = 4\frac{1}{2} < x_B$ dus

B ligt rechts van A

5C $3\sin(x) - 2\sin^2(x) = 1$

$2\sin^2(x) - 3\sin(x) + 1 = 0$

$D = 9 - 4 \cdot 2 \cdot 1 = 1$

$\sin(x) = \frac{3 \pm 1}{4}$

$\sin(x) = \frac{1}{2} \lor \sin(x) = 1$

$x = \frac{1}{6}\pi + k2\pi \lor x = \frac{5}{6}\pi + k2\pi \lor$

$x = \frac{1}{2}\pi + k2\pi$

$0 \le x \le \pi$ geeft $x_A = \frac{1}{6}\pi \lor x_B = \frac{5}{6}\pi$

$AB = \frac{5}{6}\pi - \frac{1}{6}\pi = \frac{2}{3}\pi$

5D $O(V) = \int_0^{\pi} (3\sin(x) - 2\sin^2(x))\,dx$

$= \int_0^{\pi} (3\sin(x) - 1 + \cos(2x))\,dx$

$= \left[-3\cos(x) - x + \frac{1}{2}\sin(2x)\right]_0^{\pi}$

$= -3\cos(\pi) - \pi + \frac{1}{2}\sin(2\pi) - (-3\cos(0) - 0 + \frac{1}{2}\sin(0))$

$= 3 - \pi + 3 = 6 - \pi$